Instructions: Try these problems from the textbook as practice (answers below).

Problems

3-8. Use the Chain Rule to compute $\frac{df}{dt}$.

3. $f = xy^3 - x^2y$, $x = t^2 + 1$, $y = t^2 - 1$
5. $f = \sin x \cos y$, $x = \sqrt{t}$, $y = 1/t$
7. $f = xe^{y/z}$, $x = t^2$, $y = 1 - t$, $z = 1 + 2t$

11-16. Use the Chain Rule to compute $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

11. $z = (x - y)^5$, $x = s^2t$, $y = st^2$
13. $z = \ln(3x + 2y)$, $x = s\sin(t)$, $y = t\cos(s)$
15. $z = (\sin\theta)/r$, $r = st$, $\theta = s^2 + t^2$

17. Suppose $f(x, y)$ is differentiable and $p(t) = f(g(t), h(t))$, $g(2) = 4$, $g'(2) = -3$, $h(2) = 5$, $h'(2) = 6$, $f_x(4, 5) = 2$, and $f_y(4, 5) = 8$. Compute $p'(2)$.

25-30. Use the Chain Rule to compute the indicated partial derivatives.

25. $z = x^4 + x^2y$, $x = s + 2t - u$, $y = stu^2$;
$\frac{\partial z}{\partial s}$, $\frac{\partial z}{\partial t}$, $\frac{\partial z}{\partial u}$ when $s = 4$, $t = 2$, and $u = 1$.
27. $w = xy + yz + zx$, $x = r\cos\theta$, $y = r\sin\theta$, $z = r\theta$;
$\frac{\partial w}{\partial r}$, $\frac{\partial w}{\partial \theta}$ when $r = 2$ and $\theta = \pi/2$.
29. $N = \frac{p + q}{p + r}$, $p = u + vw$, $q = v + uw$, $r = w + uv$;
$\frac{\partial N}{\partial u}$. $\frac{\partial N}{\partial v}$, $\frac{\partial N}{\partial w}$ when $u = 2$, $v = 3$, $w = 4$

39. The temperature at a point (x, y) is $T(x, y)$, measured in degree Celsius. A bug crawls so that its position after t seconds is given by $x = \sqrt{1 + t}$, $y = 2 + \frac{1}{3}t$, where x and y are measured in centimetres. The temperature function satisfies $T_x(2, 3) = 4$ and $T_y(2, 3) = 3$. How fast is the temperature rising on the bug's path after 3 seconds?

45. The pressure of 1 mole of an ideal gas is increasing at a rate of 0.05kPa/s and the temperature is increasing at a rate of 0.15K/s. Use the equation $PV = 8.31T$ to find the rate of change of the volume when the pressure is 20kPa and the temperature is 320K.

47. One side of a triangle is increasing at a rate 3 cm/s and a second side is decreasing at a rate of 2 cm/s. If the area of the triangle remains constant, at what rate does the angle between the sides change when the first side is 20cm long, the second side is 30cm, and the angle is $\pi/6$ radians?

49. If $z = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$. Show that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$.

Answers

3. $\dfrac{df}{dt} = ((t^2-1)^3 - 2(t^2+1)(t^2-1))(2t) + (3(t^2+1)(t^2-1)^2 - (t^2+1)^2)(2t)$

5. $\dfrac{df}{dt} = \cos(\sqrt{t})\cos(1/t)(\frac{1}{2}t^{-\frac{1}{2}}) - \cos(\sqrt{t})\cos(1/t)(-t^{-2})$

7. $\dfrac{df}{dt} = \exp\left(\dfrac{1-t}{1+2t}\right)(2t) + t^2\exp\left(\dfrac{1-t}{1+2t}\right)(-1) + t^2\exp\left(\dfrac{1-t}{1+2t}\right)(2)$

11. $\frac{\partial z}{\partial s} = 5(s^2t - st^2)^4(2st) - 5(s^2t - st^2)^4(t^2)$,
$\frac{\partial z}{\partial t} = 5(s^2t - st^2)^4(s^2) - 5(s^2t - st^2)^4(2st)$

13. $\dfrac{\partial z}{\partial s} = \dfrac{3}{2s\sin(t) + 2t\cos(s)}(\sin(t)) + \dfrac{2}{2s\sin(t) + 2t\cos(s)}(-t\sin(s))$,
$\dfrac{\partial z}{\partial t} = \dfrac{3}{2s\sin(t) + 2t\cos(s)}(s\cos(t)) + \dfrac{2}{2s\sin(t) + 2t\cos(s)}(\cos(s))$

15. $\frac{\partial z}{\partial s} = -\sin(s^2+t^2)(st)^{-2}(t) + \cos(s^2+t^2)(st)^{-1}(2s)$,
$\frac{\partial z}{\partial t} = -\sin(s^2+t^2)(st)^{-2}(s) + \cos(s^2+t^2)(st)^{-1}(2t)$

17. $p'(2) = 42$

25. $z_s(4,2,1) = 1526$,
$z_t(4,2,1) = 2909$,
$z_u(4,2,1) = -1036$

27. $w_r(2, \frac{\pi}{2}) = 2\pi$,
$w_\theta(2, \frac{\pi}{2}) = -2\pi$

29. $N_u(2,3,4) = -\frac{1}{8}$,
$N_v(2,3,4) = -\frac{53}{576}$,
$N_w(2,3,4) = -\frac{13}{288}$

39. 2C/s

45. decreasing by $\dfrac{263427}{40000}\text{m}^3/\text{s}$